

## **Title: Investigating Population Growth**

### **Brief Overview:**

Students will collect, organize, represent and interpret data on population size. They will make predictions based on these data and compare exponential and logistic population growth statistical models. Students will calculate descriptive statistics, draw graphs, and obtain regression models using the TI-83 calculator.

### **NCTM 2000 Principles for School Mathematics:**

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

### **Links to NCTM 2000 Standards:**

- **Content Standards**

- **Number and Operations**

- Students will judge the reasonableness of numerical computations and their results.

- **Algebra**

- Students will generalize patterns using explicitly-defined functions. They will understand and compare the properties of classes of functions, including exponential functions.

- **Measurement**

- Students will make decisions about units and scales that are appropriate for problem situations involving measurement.

### **Data Analysis and Probability**

Students will understand the meaning of measurement data, univariate and bivariate data, and the term variable. They will understand scatterplots and use them to display data. Students will compute basic statistics and understand the distinction between a statistic and a parameter.

Students will be able to display the distribution, describe its shape, and calculate summary statistics for univariate measurement data. For bivariate measurement data, they will be able display a scatter plot, describe its shape, and determine regression coefficients and regression equations. Students will identify trends in bivariate data and find functions that model the data.

### **• Process Standards**

#### **Problem Solving**

Students will build new mathematical knowledge through solving problems related to real world population growth situations.

#### **Reasoning and Proof**

Students will make conjectures based on data and they will draw conclusions based on a comparison of models.

#### **Communication**

Students will communicate their understanding numerically, graphically, symbolically and verbally. Students will work cooperatively in groups to discuss and compare their results.

#### **Connections**

Students will connect ideas in these lessons to other real life situations and to content addressed in other fields such as biology and history.

#### **Representation**

Students will represent data graphically, numerically, and symbolically.

### **Links to Maryland High School Mathematics Core Learning Units:**

#### **Functions and Algebra**

##### **• 1.1.2**

The student will represent patterns and/or functional relationships in a table, as a graph, and/or by mathematical expression.

##### **• 1.1.3**

The student will apply addition, subtraction, multiplication, and/or division of algebraic expressions to mathematical and real-world problems.

- **1.1.4**

The student will describe the graph of a non-linear function and discuss its appearance in terms of the basic concepts of rate of change.

- **1.2.5**

The student will apply formulas to solve real-world problems.

**Data Analysis and Probability**

- **3.1.1**

The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

- **3.1.2**

The student will use the measures of central tendency and/or variability to make informed conclusions.

- **3.2.1**

The student will make informed decisions and predictions based upon the results of simulations and data from research.

- **3.2.2**

The student will interpret data and/or make predictions by finding and using a line of best fit and by using a given curve of best fit.

- **3.2.3**

The student will communicate the use and misuse of statistics.

**Links to National Science Education Standards:**

- **Unifying Concepts and Processes**

Students should develop an understanding of the concepts of evidence, models, and explanation.

- **Science as Inquiry**

Students will participate in scientific investigations and formulate scientific explanations. They will reflect on the concepts that guide the inquiry. Students also will analyze evidence and data and communicate their explanations.

- **Life Science**

Students will understand the relationship of data through the use of technology. They will be able to develop accurate concepts of the role and the relationship of science with technology interaction.

- **Science in Personal and Social Perspectives**

Students will develop an understanding of population growth and the capacity of technology to improve environmental quality.

**Links to Maryland High School Science Core Learning Units:**

**Skills and Processes**

- **1.4.1**

The student will organize data appropriately using techniques such as tables, graphs, and webs (for graphs: axes labeled with appropriate quantities, appropriate units on axes, axes labeled with appropriate intervals, independent and dependent variables on correct axes, appropriate title.

- **1.4.2**

The student will analyze data to make predictions, decisions, or draw conclusions.

- **1.4.4**

The student will determine the relationships between quantities and develop the mathematical model that describes these relationships.

- **1.4.6**

The student will describe trends revealed by data.

- **1.5.3**

The student will use computers and/or graphing calculators to produce the visual materials (tables, graphs, and spreadsheets) that will be used for communicating results.

- **1.6.2**

The student will use computers and/or graphing calculators to perform calculations for tables, graphs, or spreadsheets.

**Concepts of Biology**

- **3.5.1**

The student will analyze the relationships between biotic diversity and abiotic factors in environments and the resulting influence on ecosystems.

- **3.6.2**

The student will investigate a biological issue and be able to defend their position on population growth.

## **Environmental Science**

### **• 6.2.3**

The student will conclude that populations grow or decline due to a variety of factors.

At least —

- Linear/exponential growth
- Factors unique to the human population (medical, agricultural, cultural)

### **Grade/Level:**

Lesson 1: Grades 7-12; Introduction to Algebra or higher

Lesson 2: Grades 9-12; Algebra II, Precalculus, IB Math Studies/Methods

Lesson 3: Grades 9-12; Biology

### **Duration/Length:**

Lesson 1: One 50-minute class period and one home assignment

Lesson 2: Two 50-minutes class periods

Lesson 3: One 50-minute period

Math assessment: One 50-minute class period

Science assessment: One 30-minute class period

### **Prerequisite Knowledge:**

Students should have working knowledge of the following skills:

- Estimating, rounding, measuring and place value
- Plotting points on a rectangular coordinate plane
- Collecting and organizing data
- Calculating descriptive statistics, including the mean and median
- Identifying graphs
- Interpreting graphical representation of data
- Defining terminology

### **Student Outcomes:**

Students will:

- Construct and make inferences from data tables and graphs.
- Gather and graphically represent data.
- Calculate descriptive statistics for data and make inferences from summary statistics.
- Use the TI-83 calculator to analyze data and evaluate statistical models of real world data.

**Materials/Resources/Printed Materials:**

- TI-83 calculator
- Teacher notes
- Student worksheets
- Resource sheets
- Assessment sheets
- Teacher answer keys

**Development/Procedures:**

The students will need to have a prior knowledge with estimating, graphing, and plotting points. The teacher will demonstrate use of the TI-83 calculator to perform basic statistical functions. The class discussion will then lead to mathematical and biological issues about population growth.

**Extension/Follow Up:**

- Students can investigate doubling time and “the rule of 72” with annual growth rate data.
- Students can read about the significance of the world population reaching six billion at <http://www.msnbc.com/new/307068.asp?cp1=1>.
- Students can conduct further research on population growth on the internet:  
<http://www.prb.org>  
<http://www.fao.org/wfs/final/WFSmaps/Map02-e.pdf>

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**References:**

U.S. Bureau of the Census, <http://www.census.gov/ftp/pub/ipc/www/world.html> ,  
(accessed June 28, 2002).

## Lesson 1: A Century of United States Population Growth

Name: \_\_\_\_\_ Date: \_\_\_\_\_

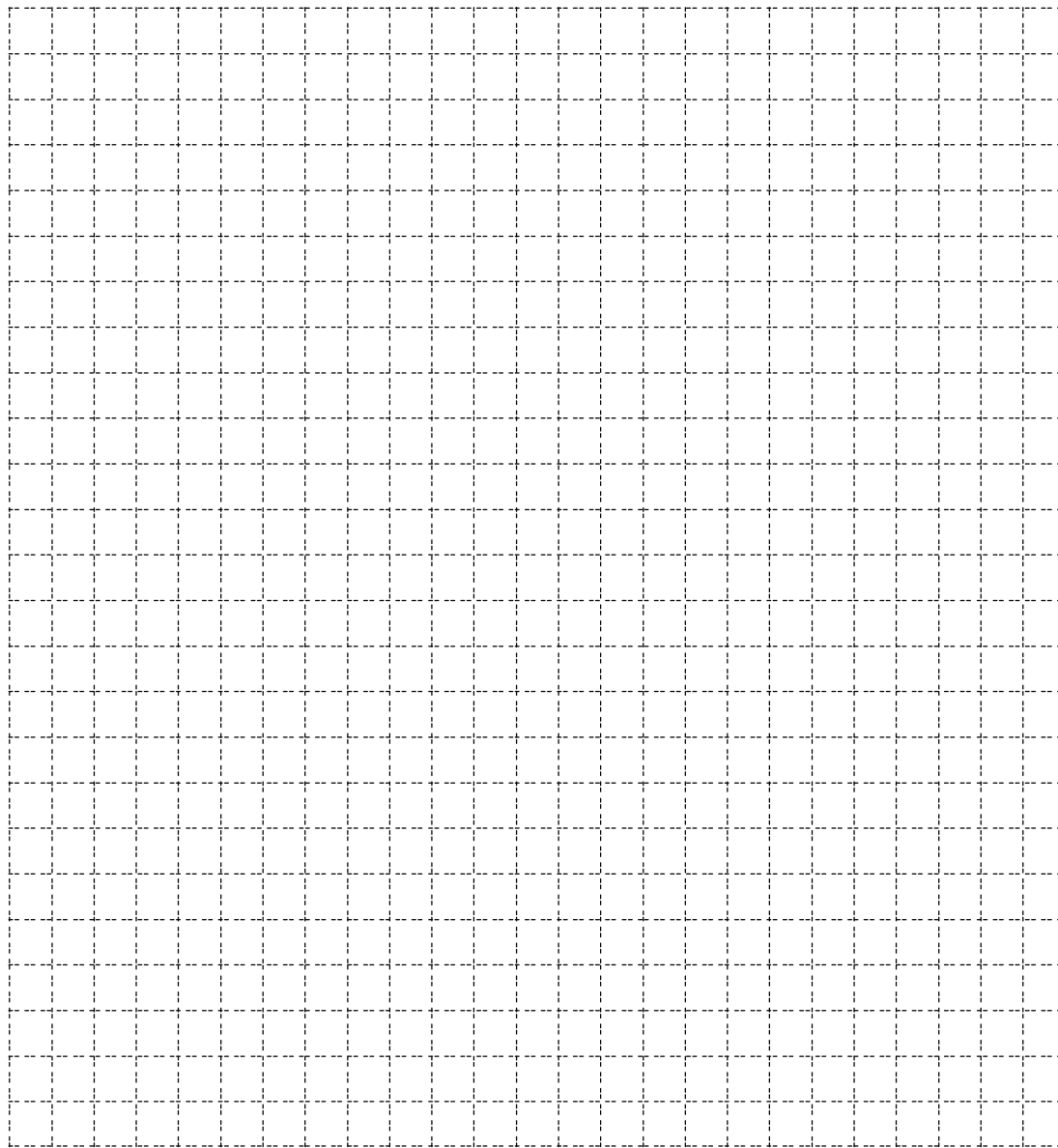
Along with the following information students will use the TI-83 calculator to answer the questions:

**The Population of the United States by the Decade 1900-2000**

Year	Population	Population rounded to the nearest million
1900	76,212,168	
1910	92,228,496	
1920	106,021,537	
1930	123,202,624	
1940	132,164,569	
1950	151,325,798	
1960	179,323,175	
1970	203,302,031	
1980	226,542,203	
1990	248,709,873	
2000	281,421,906	

1. Complete the chart by rounding the population to the nearest millions.
2. Enter the year into list 1. Use only the last two numbers of the year. Since 1900 is 0, how would you enter the year 2000? \_\_\_\_\_
3. Enter the rounded population in millions in list 2.
4. Use the TI-83 to graph information entered in list 1 and list 2. What information should be listed on the X-axis? \_\_\_\_\_
5. Which information should be listed on the Y-axis? \_\_\_\_\_

6. Show your answer on the grid.



7. How does your drawing compare to that on the calculator?

8. The average number of Americans per household is 2.59. Survey 20 of your friends to find out how their households compare to that average.

Family Number	Number of people/household
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

9. Enter the number of people per household into list 1.

10. Use the calculator to find the mean and the 5-number summary ( minimum, 1<sup>st</sup> quartile, median, 3<sup>rd</sup> quartile, and maximum) of the population data. Complete the chart:

Mean	Minimum	1 <sup>st</sup> Quartile	Median	3 <sup>rd</sup> Quartile	Maximum

11. How does the average number of people per household for the US compare to that of the people that you surveyed? Is it greater or less than the national average? \_\_\_\_\_
12. Write a paragraph discussing how this relates to the population of your neighborhood.

## Lesson 2: A Millennium of World Population Growth

Name: \_\_\_\_\_ Date: \_\_\_\_\_.

### Data Sources:

<http://www.census.gov/ftp/pub/ipc/www/world.html>

Historical Estimates of World Population from the U.S. Census Bureau

The lesson uses the average of the lower and upper estimates for years 1000-1940.

World Population: 1950 to 2050 from the U.S. Census Bureau

The lesson uses the midyear population values for each decade 1950-2050.

### Procedures:

Form small groups (2-4 students) so you can check your calculations and discuss your results.

**Table 1** summarizes world population values from 1000 A.D. to the year 2000.

**Table 1: World Population Over 1000 Years**

Year (A.D.)	Population (in millions) <sup>†</sup>
1000	300
1100	311
1200	405
1300	396
1400	362
1500	483
1600	562
1700	640
1750	795
1800	969
1850	1265
1900	1656
1910	1750
1920	1860
1930	2070
1940	2300
1950	2555
1960	3039
1970	3708
1980	4457
1990	5284
2000	6080

<sup>†</sup>Example: The 1950 world population reported as 2555 millions means that there were approximately 2,555,000,000 people.

1. Calculate the average number of people added to the world per year.  
Record your results in **Table 2** (column 2).

Formula:  $\text{Average} = \frac{\text{Change in population}}{\text{time period}}$

Example: Calculate the average number of people added 1940 to 1950

$$\begin{aligned}\text{Average} &= \frac{1950 \text{ population} - 1940 \text{ population}}{\text{time period}} \\ &= \frac{2555 - 2300}{10 \text{ years}} \\ &= 25.5 \frac{\text{million people}}{\text{year}}\end{aligned}$$

2. Each value you calculated above represents the slope (gradient) of the line that connects two consecutive data points. This assumes that growth of the world population is linear (constant) over that time period.

Comment on the validity of this assumption made for times periods with durations of 10, 50, or 100 years.

3. Calculate the average growth rate (%) per century using the data in **Table 1**. Record your results in **Table 2** (column 3).

Formula:  $\text{Growth Rate} = \frac{\text{new population} - \text{old population}}{\text{old population}} \times 100\%$

Example: Calculate the population growth rate over 1700-1800

$$\begin{aligned}\text{Growth Rate} &= \frac{969 - 640}{640} \times 100\% \\ &= 51.4\%\end{aligned}$$

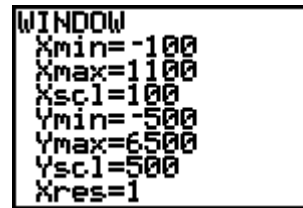
**Table 2: Growth in World Population Over 1000 Years**

Year (A.D.)	Average Number of People Added Per Year (millions)	Population Growth Rate Per Century (%)
1000		
1000-1100		
1100-1200		
1200-1300		
1300-1400		
1400-1500		
1500-1600		
1600-1700		
1700-1750		
1750-1800		
1800-1850		
1850-1900		
1900-1910		
1910-1920		
1920-1930		
1930-1940		
1940-1950		
1950-1960		
1960-1970		
1970-1980		
1980-1990		
1990-2000		

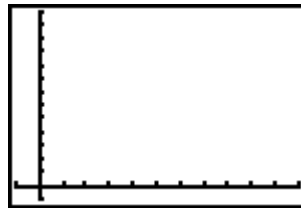
4. Describe the general trend in world population growth that you observe from the data and your calculations.
5. Relate historical and biological events to the changes in world population growth. Discuss the following questions with your group members.
  - (a) From 1200 to 1400 there was a negative period of population growth. State a reason for this observation.
  - (b) Find the ratio of the growth rates for 1700-1800 to 1600-1700? \_\_\_\_\_.  
Give a reason for this dramatic change in population growth rate.
  - (c) In the decade 1950-1960, the average number of people added per year nearly doubles. Give two distinct reasons for this sudden increase.

6. Carry out a graphical analysis of world population data over 1000 years.

- (a) Enter the data in **Table 1** into lists L1 (year) and L2 (population) on your TI-83 calculator. Enter the years as 0 = 1000 A.D., 100 = 1100 A.D., ..., 1000 = 2000 A.D. Refer to **Resource 1** if needed.
- (b) Obtain a scatter plot of the population (y-axis) versus (x-axis) on the TI-83 calculator. Make the following selections in **STATPLOT** (left screen). Alter the **WINDOW** as indicated (right screen). To see the scatter plot, press **GRAPH** (do not press ZOOM 9 in this case).



Sketch the graph you observe in the calculator screen view below.  
Label the axes.



- (c) Comment on the shape of the graph.

7. Population specialists have made projections about future population growth based on recent trends. **Table 3** summarizes recent and projected values for the years 1950-2050.

**Table 3: Recent and Projected World Population, 1950-2050**

Year (A.D.)	Population (in millions)
1950	2555
1955	2780
1960	3039
1965	3346
1970	3708
1975	4088
1980	4457
1985	4855
1990	5284
1995	5691
2000	6080
2005	6461
2010	6824
2015	7176
2020	7518
2025	7841
2030	8140
2035	8417
2040	8668
2045	8897
2050	9104

- (a) Enter the data in **Table 3** into lists L1 (year) and L2 (population) on your TI-83 calculator. Enter the years as 50 = 1950 A.D., 55 = 1955 A.D.,..., 150 = 2050 A.D. Refer to **Resource 1** if needed.
- (b) Obtain an exponential regression model for the data with the following steps on the TI-83 calculator:

**STAT/CALC/0:ExpReg/ENTER/  
L1/ , /L2/ , /VARS/Y-VARS/1:Function/Y1/ENTER/ENTER**

Record the parameter values of the regression model in the form:

$$y = a * b^x \quad a = \underline{\hspace{2cm}} \quad b = \underline{\hspace{2cm}}.$$

- (c) Observe the fit of the exponential model to the data with the following steps. Graph the data on a scatter plot on the TI-83 calculator. Refer to **Resource 1** is needed. When you press **ZOOM9:ZoomStat** the calculator will plot the data points and overlay the graph with the exponential model you just found.

Comment on the fit of the exponential model to the data on recent and project world population, 1950-2050.

8. (a) Obtain a logistic regression model for the data with the following steps on the TI-83 calculator:

**STAT/CALC/B:Logistic/ENTER/  
L1/ , /L2/ , /VARS/Y-VARS/1:Function/Y1/ENTER/ENTER**

Record the parameter values of the regression model in the form:

$$y = \frac{c}{1 + ae^{-bx}} \quad c = \underline{\hspace{2cm}} \quad b = \underline{\hspace{2cm}}$$

- (b) Observe the fit of the logistic model to the data. Since you have just plotted the data on a scatter plot in Step 7 above, you should only need to press **GRAPH**.

Comment on the fit of the logistic model to the data on recent and project world population, 1950-2050.

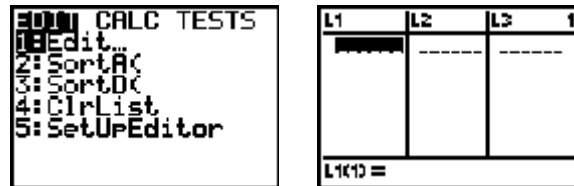
- (c) Explain the meaning of the parameters  $c$  and  $b$ .

## Investigating Population Growth

### Resource 1: Basic Statistics on the TI-83 Calculator

To enter data into a list:

- Turn the calculator ON.
- Press **STAT/EDIT/1: Edit/ENTER**



- Type each entry in L1. Press **ENTER** after each entry.
- Use the **arrow keys** to move to L2. Enter a second set of data into L2.

(Top of calculator screen)

L1	L2	L3	2
0	76	-----	
10	82		
20	106		
30	123		
40	122		
50	151		
60	179		
L2(1)=76			

(Table continues as you scroll down.)

L1	L2	L3	2
50	151		
60	179		
70	203		
80	227		
90	249		
100	281		
L2(12)=			

To obtain a scatter plot of the data:

Press <b>2<sup>nd</sup>/STAT PLOT / 1: Plot /ENTER</b>		Select the options indicated below.	

Press <b>ZOOM/9: ZoomStat</b>	Observe the scatter plot.	Press <b>TRACE</b> to see the paired coordinates you entered into L1 and L2.

**To find the mean and the 5-number summary:**

- Enter data into L1 as done before.

L1	L2	L3	1
0			
10			
15			
7			
8			
6			
3			

L1(1) = 0

Press <b>STAT</b>		Highlight <b>CALC</b>		Press <b>ENTER</b>	
1: <b>EDIT...</b>		1: <b>1-Var Stats</b>		1: <b>1-Var Stats</b>	
2: <b>SortA(</b>		2: <b>2-Var Stats</b>			
3: <b>SortD(</b>		3: <b>Med-Med</b>			
4: <b>ClrList</b>		4: <b>LinReg(ax+b)</b>			
5: <b>SetUpEditor</b>		5: <b>QuadReg</b>			
		6: <b>CubicReg</b>			
		7: <b>QuartReg</b>			

- Press **ENTER**

(Top of statistics results)

1-Var Stats  
 $\bar{x}=4.6$   
 $\Sigma x=92$   
 $\Sigma x^2=706$   
 $Sx=3.85800603$   
 $\sigma x=3.760319135$   
 $\downarrow n=20$

(Scroll down screen to view more results.)

```
1-Var Stats
n=20
minX=0
Q1=1.5
Med=4
Q3=6.5
maxX=15
```

(Key to 1-Var Stats or one variable statistics)

$\bar{x}$  = mean (average)

**$n$  = number of entries in list**

**min X = minimum number**

**Q<sub>1</sub> = 1<sup>st</sup> quartile**

**Med = median**

**$Q_3 = 3^{\text{rd}}$  quartile**

**max X = maximum number**

## ANSWER KEY

### Lesson 1: A Century of United States Population Growth

Along with the following information students will use the TI-83 calculator to answer the questions:

**The Population of the United States by Decade 1900 to 2000**

Year	Population	Population rounded to the nearest million
1900	76,212,168	<b>76</b>
1910	92,228,496	<b>92</b>
1920	106,021,537	<b>106</b>
1930	123,202,624	<b>123</b>
1940	132,164,569	<b>132</b>
1950	151,325,798	<b>151</b>
1960	179,323,175	<b>179</b>
1970	203,302,031	<b>203</b>
1980	226,542,203	<b>227</b>
1990	248,709,873	<b>249</b>
2000	281,421,906	<b>281</b>

- Complete the chart by rounding the population to the nearest millions.
- Enter the year into list 1. Use only the last two numbers of the year. Since 1900 is 0, how would you enter the year 2000? **100**

L1	L2	L3	1
0	---	---	
10	---	---	
20	---	---	
30	---	---	
40	---	---	
50	---	---	
60	---	---	
L1(1)=0			

L1	L2	L3	1
50			
60			
70			
80			
90			
100			
L1(5)=40			

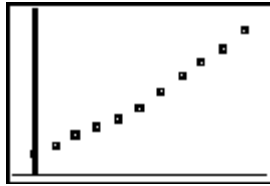
- Enter the rounded population in millions in list 2.

L1	L2	L3	2
0	76	---	
10	92	---	
20	106	---	
30	123	---	
40	132	---	
50	151	---	
60	179	---	
L2(1)=76			

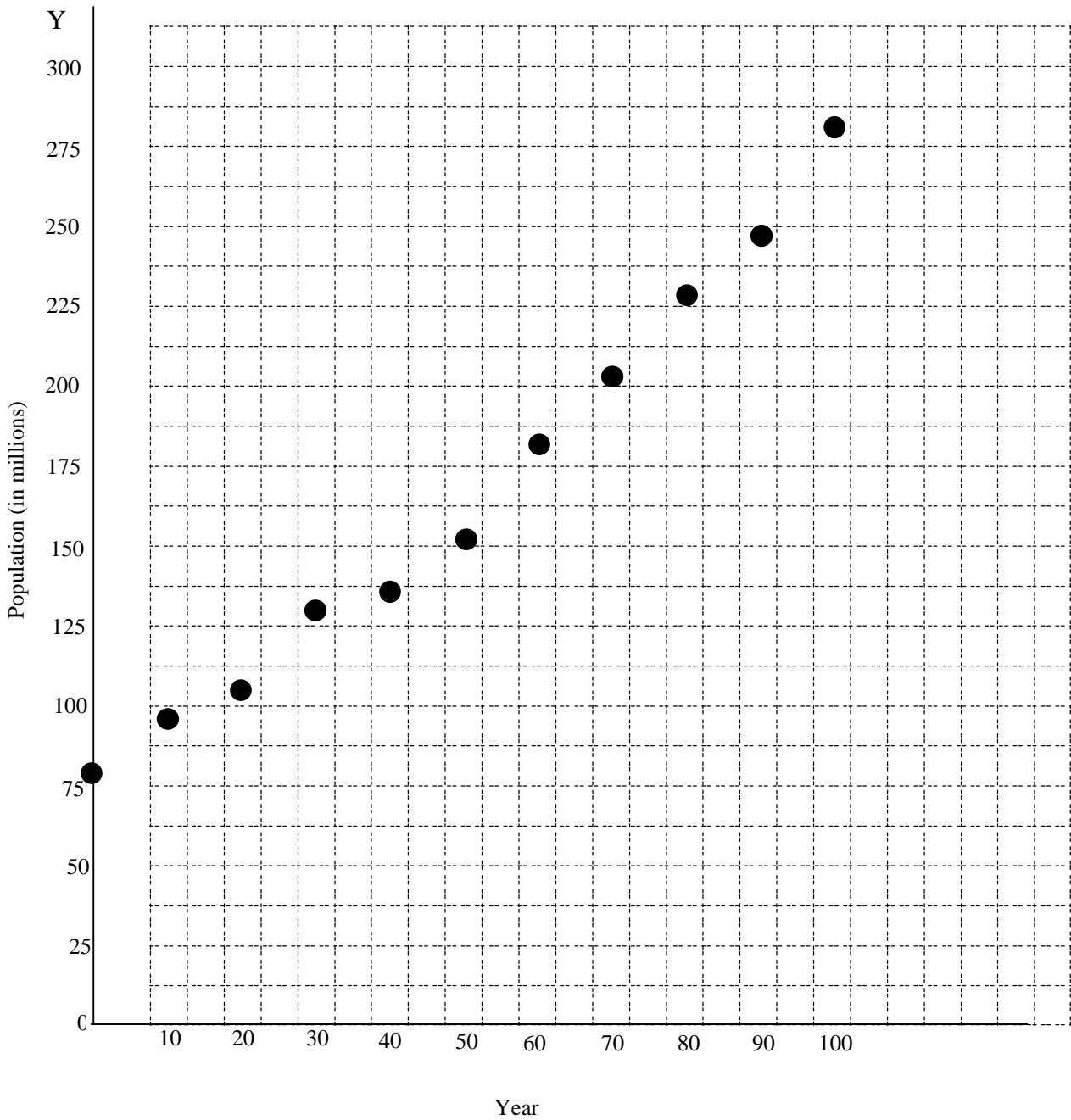
L1	L2	L3	2
50	151		
60	179		
70	203		
80	227		
90	249		
100	281		
L2(12)=			

- Use the TI-83 to graph information entered in list 1 and list 2. What information should be listed on the X-axis? **year**

5. Which information should be listed on the *Y*-axis? **rounded population in the millions**



6. Show your answer on the grid.



7. How does your drawing compare to that on the calculator?  
**The calculator does not show the scale. Answers will vary.**

8. The average number of Americans per household is 2.59. Survey 20 of your friends to find out how their households compare to that average. (**Answers will vary.**)

Family Number	Number of people/household
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

9. Enter the number of people per household into list 1. (**Answers will vary.**)

10. Use the calculator to find the mean and the 5-number summary (minimum, 1<sup>st</sup> quartile, median, 3<sup>rd</sup> quartile, and maximum) of the population data. Complete the chart: (**Answers will vary.**)

Mean	Minimum	1 <sup>st</sup> Quartile	Median	3 <sup>rd</sup> Quartile	Maximum

11. How does the average number of people per household for the US compare to that of the people that you surveyed? Is it greater or less than the National average? (**Answers will vary.**)

12. Write a paragraph discussing how this relates to the population of your neighborhood? (**Answers will vary.**)

## ANSWER KEY

### Lesson 2: A Millennium of World Population Growth

Questions 1 and 3.

**Table 2: Growth in World Population Over 1000 Years**

Year (A.D.)	Average Number of People Added Per Year (millions)	Population Growth Rate Per Century (%)
1000	--	--
1000-1100	<b>0.11</b>	<b>3.7</b>
1100-1200	<b>0.94</b>	<b>3.0</b>
1200-1300	<b>- 0.09</b>	<b>- 2.2</b>
1300-1400	<b>- 0.34</b>	<b>- 8.6</b>
1400-1500	<b>1.21</b>	<b>33.4</b>
1500-1600	<b>0.79</b>	<b>16.4</b>
1600-1700	<b>0.78</b>	<b>13.9</b>
1700-1750	<b>3.10</b>	--
1750-1800	<b>3.48</b>	<b>51.4</b>
1800-1850	<b>5.92</b>	--
1850-1900	<b>7.82</b>	<b>70.9</b>
1900-1910	<b>9.40</b>	--
1910-1920	<b>11.00</b>	--
1920-1930	<b>21.00</b>	--
1930-1940	<b>23.00</b>	--
1940-1950	<b>25.50</b>	--
1950-1960	<b>48.40</b>	--
1960-1970	<b>66.90</b>	--
1970-1980	<b>74.90</b>	--
1980-1990	<b>82.70</b>	--
1990-2000	<b>79.60</b>	<b>267.1</b>

2. Comment on the validity of this assumption made for times periods with durations of 10, 50, or 100 years.

**Although population growth may be fairly steady (linear) over short periods of time (perhaps 10 years), there is little reason to think that the growth rate remains the same over longer periods of time (50-100 years), especially since large population growth is observed over some time intervals.**

4. Describe the general trend in world population growth that you observe from the data and your calculations.

**The world population growth rate appears to increase over time. That is, more and more people are added over shorter periods of time in recent years.**

5. (a) From 1200 to 1400 there was a negative period of population growth. State a reason for this observation.

**The Bubonic Plague (the Black Death) spread from central Asia into Europe, killing nearly one-quarter of the population.**

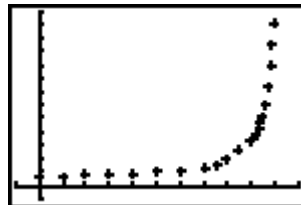
(b) Find the ratio of the growth rates for 1700-1800 to 1600-1700? 3.7 to 1.  
Give a reason for this dramatic change in population growth rate.

**The Industrial Revolution led to improved living conditions and thus more people survived and reproduced or people were able to have and support larger families.**

(c) In the decade 1950-1960, the average number of people added per year nearly doubles. Give two distinct reasons for this sudden increase.

**Two reasons for the sudden increase in 1950-1960 are (1) a baby boom after World War II and (2) better medicines, including the use of antibiotics that were discovered during the WWII era.**

6. (b) Scatter plot of the world population over 1000 years.



(c) Comment on the shape of the graph.

**The graph grows very slowly for a long time (about 800 years) and then it grows very fast over a short period of time (about 200 years).**

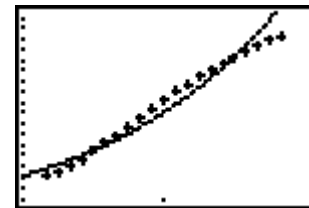
7. (b) Record the parameter values of the regression model in the form:

$$y = a * b^x \quad a = \underline{1512} \quad b = \underline{1.01}.$$

```
ExpReg
y=a*b^x
a=1512.4202
b=1.013048
r^2=.9609453506
r=.9802782006
```

(c) Comment on the fit of the exponential model to the data on recent and project world population, 1950-2050.

**The graph overestimates the population size from 1950-1965 and from 2030-2050. The graph suggests a much higher population size at 2050 than experts have projected. The exponential model also underestimates the population for most of the period 1965-2030.**



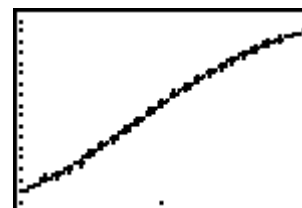
8. (a) Record the parameter values of the regression model in the form:

$$y = \frac{c}{1 + ae^{-bx}} \quad c = \underline{10676} \quad b = \underline{0.0297}.$$

```
Logistic
y=c/(1+ae^(-bx))
a=14.79381436
b=.0297039694
c=10676.43403
```

(b) Comment on the fit of the logistic model to the data on recent and project world population, 1950-2050.

**The logistic model shows a much better fit to the recent and project world population data throughout the entire time period, 1950-2050. The graph emphasizes that the population appears to begin to level off toward the end.**



(c) Explain the meaning of the parameters  $c$  and  $b$ .

**The parameter  $c$  represents the maximum population size. The parameter  $b$  represents the population growth rate.**

## **Investigating Population Growth: Performance Assessment**

### **Teacher's Guide**

#### **Introduction**

This assessment is intended to follow completion of Lessons 1 and 2 on mathematics. There are two sections to the assessment that may be combined or administered separately depending on the needs of the teacher or the level of the students. Part 1 consists of selected response items and brief constructed response questions. Part 2 is a performance activity. Part 2 may be given as a home assignment.

#### **Objectives Covered**

This assessment incorporates the objectives that students have followed in the lessons.

Students will be able to:

- Construct and make inferences from data tables and graphs.
- Gather and graphically represent data.
- Calculate descriptive statistics for data and make inferences from summary statistics.
- Use the TI-83 calculator to analyze data and evaluate statistical models of real world data.

#### **Tools/Materials Needed for Assessment**

Copy of the Student Response Sheet, pencil, ruler, TI-83 graphing calculator

#### **Administering the Assessment**

Part 1 of the assessment should take approximately 20 minutes to administer. Part 2 should take approximately 25 minutes to administer.

Give each student a copy of the Student Response Sheet – Part One.

*SAY: Today you are going demonstrate the skills you have learned and practiced in the lessons on population growth. You will work on the questions independently. First, take some time to read through the questions.*

Pause for a few moments to allow students to read through the assessment.

*SAY: Are there any questions?...You will have 20 minutes to complete Part One. You will need your TI-83 graphing calculator to answer questions. You may begin now.*

If Part Two of the assessment is to be given in class, give each student a copy of the Student Response Sheet – Part Two.

*SAY: Now, you are going complete a longer performance assessment. You will work on the questions on your own. First, take some time to read through the questions.*

Pause for a few moments to allow students to read through the assessment.

*SAY: Are there any questions?...You will have 25 minutes to complete Part Two. You will need your TI-83 graphing calculator to answer questions. You may now begin.*

After each part of the assessment, when time is completed, collect all student materials.

**Performance Assessment**  
Mathematics Assessment- Part One - Population Growth

*Student Response Sheet*

Name: \_\_\_\_\_

Date: \_\_\_\_\_

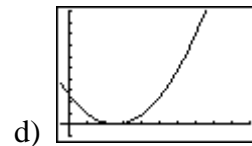
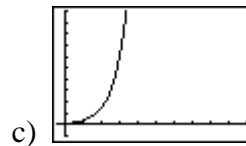
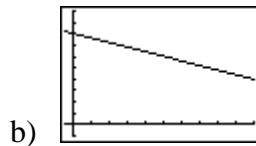
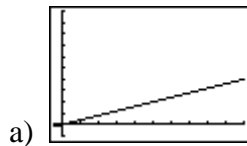
For questions 1-4 circle the correct answer.

Use the table to answer questions 1 and 2:

**Estimated Population of American Colonies, 1630-1720**

Year	Population (in thousands)
1630	4.6
1650	50.4
1670	111.9
1690	210.4
1700	250.9
1720	466.2

1. The population in 1690 was:  
a) 210,400                      b) 2,104                      c) 214,000                      d) 21,040
2. If you graphed the data in the chart, how would you label the  $x$ -axis?  
a) Population (in thousands)                      b) Population                      c) Year                      d) 1720
3. 1656 million can be written out as  
a) 1,656,000                      b) 1,656,000,000                      c) 165,600,000                      d) 165,600
4. Which graph represents an exponential growth?



Use the table above to answer question 5:

5. Find the mean and the 5-number summary for the Estimated Population of American Colonies:

Mean	Minimum	1 <sup>st</sup> Quartile	Median	3 <sup>rd</sup> Quartile	Maximum

6. The world population today is 6234 million (to the nearest million) and it was 4613 million in 1982. Calculate the population growth rate over the past 10 years. Show all steps in your working out.

7. Growth in a population is described by the equation  $y = 15 * 2^x$ .

- (a) Find the value of  $y$  when  $x = 4$  years;
- (b) Find the value of  $y$  when  $x = 5$  years.
- (c) Use your answers to parts (a) and (b) to **explain** what the **2** in the equation represents.

**Performance Assessment**  
Mathematics Assessment- Part Two - Population Growth

*Student Response Sheet*

**Name:** \_\_\_\_\_

**Date:** \_\_\_\_\_

**Read the following passage and answer the questions that follow.**

A new apartment building opened in Baltimore, Maryland on January 1, 2001. The data in the table show how many apartments were rented by the end of selected months for the year 2001.

The bank officials that loaned money to the owners to purchase the building were worried about their investment because less than 200 apartments were rented by the end of the first year.

The owners, on the other hand, were optimistic that the growth of rented apartments would continue and that the building would be full by the end of June 2002.

**Total Apartments Rented by the End of Each Month during 2001.**

Month	Number of Apartments Rented
February	32
April	46
June	66
August	95
October	136
December	196

- (a) Comment on the growth in the number of apartments rented during the first year.
- (b) Do you think that the owners will reach their goal of a fully rented building by the end of June 2002? If no, how close do you think they will be to their goal?
- (c) Enter the data in the table in your TI-83 calculator in lists L1 (month) and L2 (number of apartments rented).
- (d) Produce a scatterplot and sketch in the calculator screen view. Make adjustments to the **WINDOW** as shown below.

```

WINDOW
Xmin=-1
Xmax=20
Xscl=1
Ymin=-10
Ymax=1200
Yscl=100
Xres=1

```



- (e) Obtain an exponential regression model for the data. Record the equation.
- (f) Graph the exponential equation you found, using the same window as in step (e).
- (g) Use the graph to decide whether the apartment will reach capacity by June 2002. Explain how you used values from your graph to make your decision.
- (h) Comment on the factors that are not recorded in the data table or accounted for by the model that may have importance in a real life situation.

## ANSWER KEY

### Performance Assessment Mathematics Assessment- Part One - Population Growth

Name: \_\_\_\_\_

Date: \_\_\_\_\_

For questions 1-4 circle the correct answer.

Use the table to answer questions 1 and 2:

**Estimated Population of American Colonies, 1630-1720**

Year	Population (in thousands)
1630	4.6
1650	50.4
1670	111.9
1690	210.4
1700	250.9
1720	466.2

1. The population in 1690 was:

- a) 210,400      b) 2,104      c) 214,000      d) 21,040

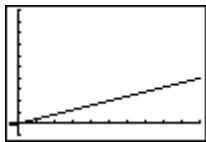
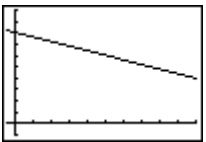
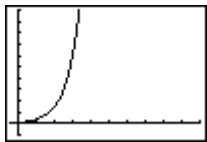
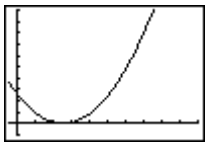
2. If you graphed the data in the chart, how would you label the x-axis?

- a) Population (in thousands)      b) Population      c) Year      d) 1720

3. 1656 million can be written out as

- a) 1,656,000      b) 1,656,000,000      c) 165,600,000      d) 165,600

4. Which graph represents an exponential growth?

- a)       b)       c)       d) 

Use the table above to answer question 5:

5. Find the mean and the 5-number summary for the Estimated Population of American Colonies:

Mean	Minimum	1 <sup>st</sup> Quartile	Median	3 <sup>rd</sup> Quartile	Maximum
<b>182.4</b>	<b>4.6</b>	<b>50.4</b>	<b>161.15</b>	<b>250.9</b>	<b>466.2</b>

6. The world population today is 6234 million (to the nearest million) and it was 4613 million in 1982. Calculate the population growth rate over the past 10 years. Show all steps in your working out.

$$\frac{6234 - 4613}{4613} \times 100\% = \frac{1621}{4613} \times 100 = 0.351 \times 100 = 35.1 \%$$

7. Growth in a population is described by the equation  $y = 15 \times 2^x$ .

(a) Find the value of  $y$  when  $x = 4$  years; **240**

(b) Find the value of  $y$  when  $x = 5$  years. **480**

(c) Use your answers to parts (a) and (b) to **explain** what the **2** in the equation represents.

**The 2 represents the growth rate in the population. Here, the population doubles every year.**

## ANSWER KEY

### Performance Assessment

#### Mathematics Assessment- Part Two - Population Growth

- (a) Comment on the growth in the number of apartments rented during the first year.

Answers may vary.

**Over the year 2001, the number of apartments rented increased. Each month, more apartments were rented than the month before.**

- (b) Do you think that the owners will reach their goal of a fully rented building by the end of June 2002? If no, how close do you think they'll be to their goal.

Answers may vary

**Yes, since the number apartments rented increases each month, the owners should be able to fill the building by June 2002.**

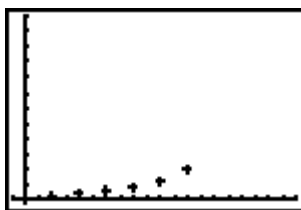
**-or-**

**No, since by the end of the first year only 196 apartments out of 1100 are rented (17.8%), it is unlikely that the building will be filled in only six more months by June 2002.**

- (c) Enter the data in the table in your TI-83 calculator in lists L1 (month) and L2 (number of apartments rented).

#### Done on the TI-83 calculator

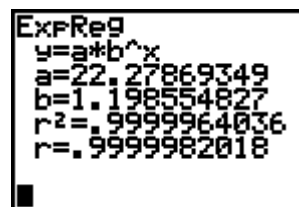
- (d) Produce a scatterplot and sketch in the calculator screen view. Make adjustments to the **WINDOW** as shown below.



- (e) Obtain an exponential regression model for the data. Record the equation.

#### Done on the TI-83 calculator

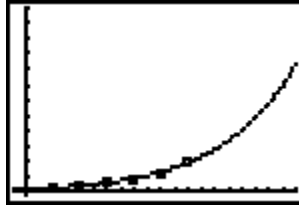
$$y = 22.3 * 1.20^x$$



```
ExpReg
y=a*b^x
a=22.27869349
b=1.198554827
r^2=.9999964036
r=.9999982018
```

(f) Graph the exponential equation you found, using the same window as step (e).

**Done on the TI-83 calculator**



(g) Use to graph to decide whether the apartment will reach capacity by June 2002. Explain how you use values from your graph to make your decision.

**By June 2002, about 580 apartments have been rented, so the building has not reached capacity. The number of apartments for June can be read as the  $y$ -value that corresponds to the  $x$ -value for June 2002 ( $x = 18$ ).**

(h) Comment on the factors that are not recorded in the data table or accounted for by the model that may have importance in a real life situation.

**The data in the table do not report whether people have moved out as well as moved in the apartment building. Some months may be better or worse times for moving. You might expect low numbers in the winter and very high numbers of apartments rented in the spring or summer when school lets out and it is easier for people to move.**

## **SCORING RUBRIC**

### **Performance Assessment**

#### **Mathematics Assessment- Part Two - Population Growth**

**3:** This student demonstrates a high ability to use the TI-83 calculator to enter data, obtain a scatter plot, and to determine the equation for an exponential model to fit a set of data. This student is also able to thoroughly and correctly interpret data in tables/graphs and make predictions based on a set of data using a statistical regression model.

**2:** This student demonstrates some ability to use the TI-83 calculator to enter data, obtain a scatter plot, and to determine the equation for an exponential model to fit a set of data. This student attempts to interpret data in tables/graphs and make some meaningful predictions based on a set of data using a statistical regression model.

**1:** This student demonstrates low ability to use the TI-83 calculator to enter data, obtain a scatter plot, and to determine the equation for an exponential model to fit a set of data. This student attempts to interpret data in tables/graphs and make predictions based on a set of data using a statistical regression model but interpretations and predictions contain some error or lack thoroughness.

**0:** This student demonstrates little or no ability to use the TI-83 calculator to enter data, obtain a scatter plot, and to determine the equation for an exponential model to fit a set of data. This student incorrectly interprets data in tables/graphs and predictions, based on a set of data using a statistical regression model, reveal a lack of understanding of concepts about regression and the fit of a model to data.

**Lesson Title: Populations and the Rate of Increase or Decrease**

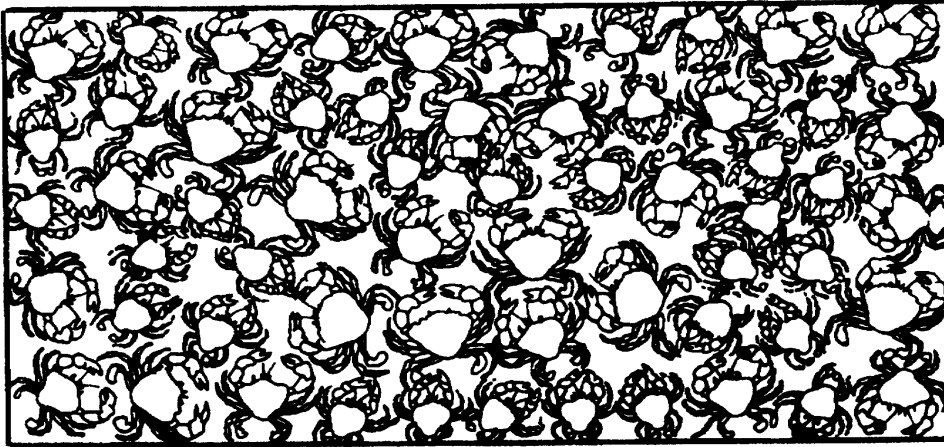
<p><b><u>Unit/Theme: Ecology/Systems and Interactions</u></b></p> <p><b><u>Quarter: 1 2 3 4</u></b></p> <p><b><u>Circle the Domain for this lesson:</u></b></p> <ul style="list-style-type: none"> <li>• Skills and Process</li> <li>• Concepts/Knowledge/Content</li> <li>• Applications</li> </ul>	<p><b><u>Expectancy:</u></b> The student will demonstrate the ability to acquire and integrate major biological concepts to explain the uniqueness and interdependence of living organisms, their interactions with the environment, and the continuation of life on earth.</p> <p><b><u>Scope and Sequence Indicators (list by number as well as statement):</u></b> The student will: <b>3.5.3 investigate how natural and man-made change in environmental conditions will affect individual organisms and the dynamics of populations including depletion of food, destruction of habitats, disease, natural disasters, pollution, population increase/decrease, and urbanization.</b></p>
<p><b><u>Approximate Time Required: 60 minutes</u></b></p> <p><b><u>Important Safety Issues: none</u></b></p> <p><b><u>Prior Knowledge/Skills Required: Familiarity with biotic and abiotic factors, names of aquatic plants, such as hydrilla, population, exponential growth, and growth with limits.</u></b></p>	<p><b><u>Needed Lab Materials:</u></b> Textbook, work sheets, graphs, science rubric, rubric test, graphing calculator, pen/pencil, and markers  <a href="http://aquat1.ifas.ufl.edu/hydver.jpg">http://aquat1.ifas.ufl.edu/hydver.jpg</a></p>
<p>How I will ENGAGE the students using connections to their prior knowledge</p>	<p><b>Have students examine diagram with captions; then have them answer the following questions:</b></p> <p><b>1.) Why do you think the hydrilla spread over such a large area?</b>          Ans. – Students should realize that the unchecked growth was due to two main factors: favorable living conditions, such as ample nutrients, and the absence of consumers that feed on hydrilla.</p> <p><b>2.) What could be done to solve the problem of hydrilla overgrowth?</b>          Ans. - Accept all reasonable responses, such as introducing a consumer species.</p> <p><b>3.) What could be done to reverse the decline in fishing catches?</b>          Ans. – Accept all reasonable responses, such as imposing limits.</p>
<p>What students will do to EXPLORE the concepts and begin to develop vocabulary at the same time.</p>	<p><b><u>Pre-Activity Discussion</u></b> - A population is a group of individuals of the same species living in a given area. Biologist can use two different methods to determine the number of living things in a given area. The most accurate data would be obtained by counting each member of the population. However, in most situations this counting method is impractical and very time consuming. A second method is to count a small sample of the population. The sample counted must be representative of the entire population. If the sample is not representative of the entire population, then the data collected are biased and therefore inaccurate. A random selection of areas in which to count organisms helps to eliminate bias. Every member of the population stands an equal chance of being counted. In this activity, students will demonstrate the technique of counting a random sample of population in comparison to counting the actual size of population.</p> <p>Introduce basic concepts developed using the learning activity on crab population study guide. Have students read about population in textbook, then follow directions as outlined on study guide by answering questions 1-3. Upon completion of the crab count activity, students will analyze population numbers by graphing the growth /decline of others organisms using a table and graph on a second study guide.</p>

<p>What I will do to allow students an opportunity to construct their own EXPLANATION of the concepts</p>	<p><b>1.) Students will construct their own explanation of concepts by measuring and analyzing collected data and data given in table and graph.</b></p> <p><b>2.) Students will be introduced to the TI-83 Calculator and will use it to input data on graphs and tables in the lesson plans.</b></p>
<p>Opportunities I will give students to ELABORATE or extend their understanding of the concepts</p>	<p><b>Students may extend their understanding of the concepts by drawing conclusions and making judgments about the following questions:</b></p> <p><b>1.) <u>Drawing conclusions:</u> Modern medicine has had a pronounced effect on both density-dependent and density-independent factors for humans. Explain how.</b> Ans. – Responses will vary. Modern medicine has had the most direct effect on density-dependent limiting factors of parasitism and crowding/stress and on the density-independent limiting factor of the effects of human activities.</p> <p><b>2.) What generally controls the birth and death rates in human? Are these factors similar to those that influence a deer population in a natural environment? Support your answer.</b></p> <p>Ans. – Responses will vary. Population control in humans is more of a conscious decision than an effect of the density-dependent limiting factors of competition, predation, and parasitism that control deer population.</p>

<p>How I and/or the student will EVALUATE his/her learning.</p>	<p><b>Using the objectives of this lesson as the basis for evaluating skill development, the attached science rubric has been developed for scoring the following questions. The rubric allows for a range of student responses. Students will employ a higher order of thinking skills by responding to the following questions:</b></p> <ol style="list-style-type: none"> <li><b>1.) Compare the density-dependent and density-independent factors that influence a population.</b> Ans.- Density-dependent limiting factors are competition, predation, parasitism, crowding, and stress. Density-independent factors are weather, fires, drought, floods, hurricanes, and human activities.</li> <li><b>2.) Construct a model that illustrates the effect of one density-independent limiting factor on a plant population.</b> Ans.- Accept all modeling responses that involve a density-independent limiting factor: weather, fire, drought, flood, or human activities.</li> <li><b>3.) Choose one of the density-dependent limiting factors and explain how it applies to human populations.</b> Ans.- Responses will vary. Initial research should focus on the pest's life cycle and on natural limiting factors in its native environment. If the researcher can find a means of interfering with the pest's life cycle – such as by introducing a host-specific parasite or disease – or can find a natural predator that is harmless to other organisms, then pesticides would not have to be used.</li> <li><b>4.) Why can we not view the human population in isolation?</b> Ans.- Human activity affects the Earth's ability to provide the essentials of life, such as food, water, air, land, shelter, and minerals.</li> <li><b>5.) What factors influence exponential growth in a typical deer population in a natural forest?</b> Ans. - Competition for resources, predation, parasitism, crowding/stress, weather, and human activities influence exponential growth in a typical deer population.</li> <li><b>6.) Why is it essential to understand human growth rates for the future of this planet?</b> Ans. - Human growth rates must be understood because the biosphere has limited resources.</li> </ol>
<p>Teacher Notes – The source of information for this lesson is the student and teacher's edition of "Biology – The Living Science" by authors Ken Miller and Joe Levine. In addition, photographs of hydrilla and fish catch were downloaded from the internet, google.com.</p>	

## POPULATIONS

1. Make a population count of the crabs in the diagram below. Place a checkmark on the shell of each crab to avoid counting any twice. Keep track of how long it takes you to do it.



Total number = \_\_\_\_\_ Time it took = \_\_\_\_\_

2. A faster way to count a population is to sample it. Count the number of crabs in the small square on the right below.

Total number = \_\_\_\_\_

Time it took = \_\_\_\_\_



This square is  $\frac{1}{8}$  the size of the large square above.  
Therefore, you need to multiply the number you counted by 8 to get the total population size.

\_\_\_\_\_  $\times 8 =$  \_\_\_\_\_

3. a. Were the results from counting about the same regardless of which method was used to count?

\_\_\_\_\_

- b. What is the advantage of counting a population by sampling it? \_\_\_\_\_

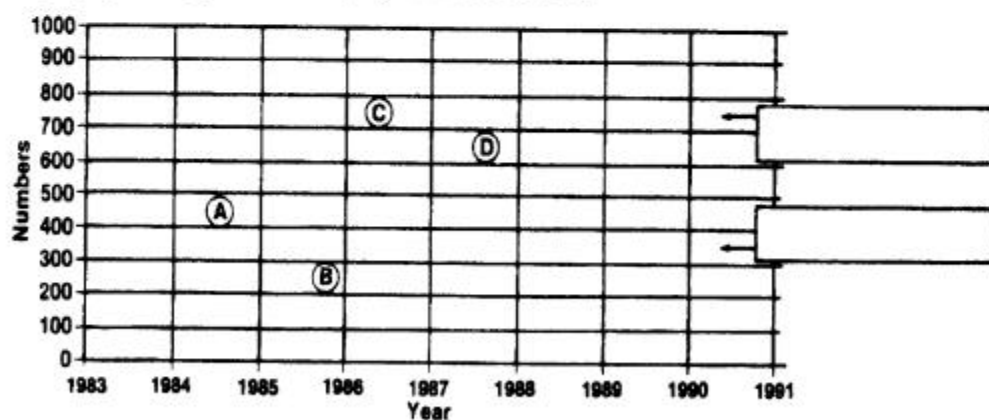
\_\_\_\_\_

## POPULATIONS

4. Many things change the numbers of individuals in a population. One important factor is food supply. For example, foxes eat mice. The table below shows how their numbers change.

Year	Number of mice	Number of foxes
1983	1050	200
1984	800	425
1985	426	581
1986	730	300
1987	980	153
1988	620	399
1989	380	548
1990	680	403
1991	1010	255

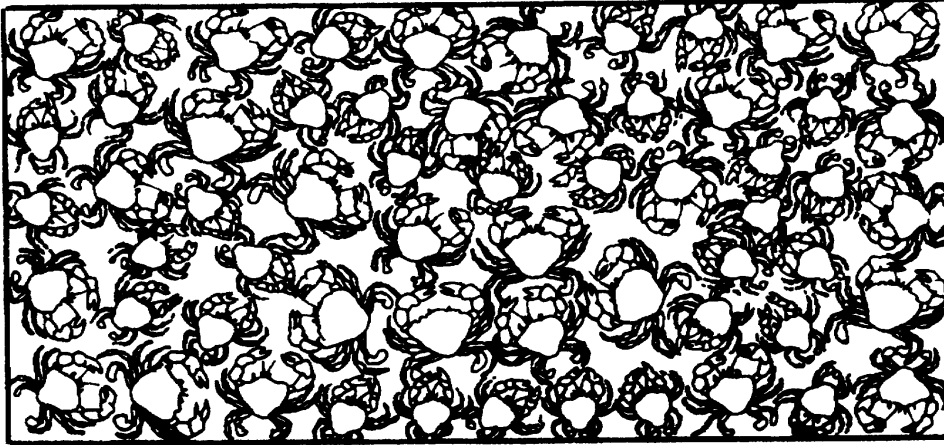
- Plot the number of mice on the graph below and connect all the points with a black line.
- Plot the number of foxes on this graph. Connect all the points with a red line.
- In the boxes at the right of the graph, indicate which animal is the predator and which is the prey by writing *predator* and *prey* in the correct box.



5. After each of the phrases below, write the letter from the graph (A, B, C, or D) that best matches.
- Fox population increasing \_\_\_\_\_
  - Fox population decreasing \_\_\_\_\_
  - Mouse population increasing \_\_\_\_\_
  - Mouse population decreasing \_\_\_\_\_

## POPULATIONS

1. Make a population count of the crabs in the diagram below. Place a checkmark on the shell of each crab to avoid counting any twice. Keep track of how long it takes you to do it.



Total number = 60 Time it took = ANSWER VARIES

2. A faster way to count a population is to sample it. Count the number of crabs in the small square on the right below.

Total number = 8  
Time it took = ANSWER VARIES



This square is  $\frac{1}{8}$  the size of the large square above.  
Therefore, you need to multiply the number you counted by 8 to get the total population size.

8  $\times$  8 = 64

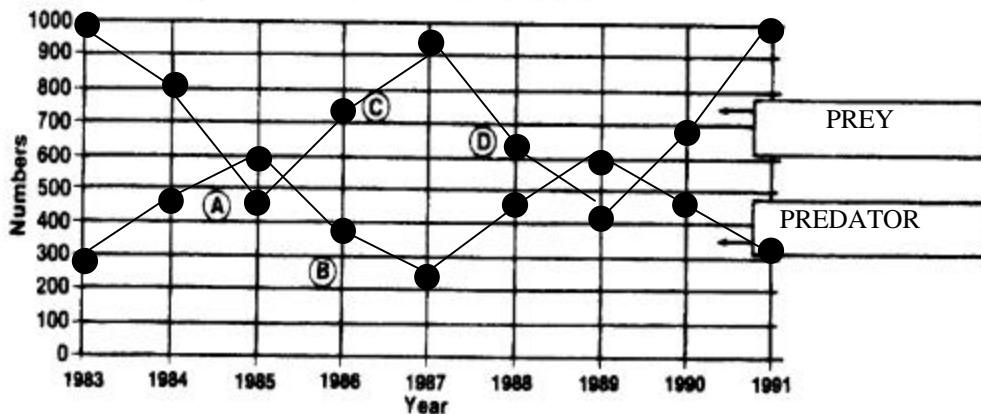
3. a. Were the results from counting about the same regardless of which method was used to count?  
YES, RESULTS WERE ALMOST THE SAME
- b. What is the advantage of counting a population by sampling it? IT IS MUCH FASTER AND ALMOST AS ACCURATE

## POPULATIONS

4. Many things change the numbers of individuals in a population. One important factor is food supply. For example, foxes eat mice. The table below shows how their numbers change.

Year	Number of mice	Number of foxes
1983	1050	200
1984	800	425
1985	426	581
1986	730	300
1987	980	153
1988	620	399
1989	380	548
1990	680	403
1991	1010	255

- Plot the number of mice on the graph below and connect all the points with a black line.
- Plot the number of foxes on this graph. Connect all the points with a red line.
- In the boxes at the right of the graph, indicate which animal is the predator and which is the prey by writing *predator* and *prey* in the correct box.



5. After each of the phrases below, write the letter from the graph (A, B, C, or D) that best matches.

- |  |  |
|--|--|
| a. Fox population increasing _____ A _____ | c. Mouse population increasing _____ C _____ |
| b. Fox population decreasing _____ B _____ | d. Mouse population decreasing _____ D _____ |